A new method of comparing forcing agents in climate models

(Supplemental Online Material)

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ABSTRACT

This document contains one section of supplemental online material and one table, all dealing with transitivity of the relationships described in Table 1 of the main text.

1. Transitivity

Transitivity as used here is taken from the mathematical concept of the same name and is an assessment of how well the regressed relationships can be combined to explain each other. As a broad example, if 2xCO_2 is equivalent to X% solar, and X% solar is equivalent to YCH_4, then is 2xCO_2 equivalent to YCH_4 according to our method? More specifically, if one knows Equations 8 and 10 (See Table 1), then how accurate is the result of combining these two equations to yield Equations 12 and 13?

In the absence of noise and any remaining nonlinearities, transitivity should hold perfectly. However, because we are recovering the regressed relationships in the presence of noise, transitivity may be challenging to recover. Additionally, as was discussed in Section 4a, relationships between certain combinations of forcing agents may be more difficult to recover than others. Our exercise here in testing transitivity is not to test whether transitivity is a fact, but rather to provide a measure of this method's ability to recover accurate relationships between the forcing agents, including both the scaling relationships and the coefficients obtained in the regressions. Section 4a gives examples of three sources of uncertainty in the explicit feedback method that could contribute to challenges in recovering these coefficients:

1. Large relative changes in the amount of a particular forcing agent are required to obtain the comparison relationship

2. Inabilities to achieve required levels of forcing, such as an inability to cause an offset (e.g., reduction in CH_4 to compensate for an increase in S_0) or reverse saturation (e.g., very low concentrations of CH_4)

3. Nonlinearities that are not related to (global mean) temperature-induced feedbacks

Here we assess the ability of our method to recover transitivity by substituting the equations in Table 1 into each other to recover other relationships in the presence of noise. We also invert the equations in Table 1. Performing all possible substitutions and inversions yields the results in the table below. Tabulation of the aggregate percent error in slope associated with each equation used in the substitution (Column 3 in the table below) yields a determination of which regressed relationships are associated with the least amount of error. When only tabulating substitutions and not inversions, the error due to Equations 8 and 10 is substantially lower than the error of Equations 12-15. Equations 12-15 can then be derived from Equations 8 and 10 via substitution or inversion. When including inversions in the tabulation, Equation 10 still has the lowest error of all equations. Of the three forcing agents in this study, using the feedback loop to change total solar irradiance will result in the most accurate comparison. This follows from the discussion elucidated in Section 4a; for example, meeting any goal of comparison with S_0 requires only small adjustments, whereas compensating adjustments in CH_4 often require very large relative changes, some of which are not even achievable.

As was described in Section 4a, the regressed relationships, and hence transitivity, are only valid within a certain range of forcings. For example, when offsetting CH_4 changes by modifying CO_2, very low values of methane cannot be captured by the regression relationship, so transitivity would not be expected for these low values.

LIST OF TABLES

Indep. Var.	Dep. Var.	Eqns.	Orig. Eqn.	Slope	Orig. Slope	Error (%)	Int.	Orig. Int.
$\log_2(XCO_2)$	XS_0	10,12	8	-0.0219	-0.0208	5.0	0.0097	-0.0010
$\sqrt{CH_4}$	XS_0	8,13	10	-0.0068	-0.0073	6.6	0.0032	0.0051
$\log_2(XCO_2)$	$\sqrt{CH_4}$	13,15	12	-2.5048	-2.9866	16.1	241.1501	0.6210
$\sqrt{CH_4}$	$\log_2(XCO_2)$	12,14	13	-0.2718	-0.3284	17.2	74.4295	0.2037
XS_0	$\log_2(XCO_2)$	10,14	14	-39.4923	-37.1243	6.4	0.4570	-0.0103
XS_0	$\sqrt{CH_4}$	8,15	15	-110.8766	-120.2520	7.8	0.5904	0.7713
$\log_2(XCO_2)$	XS_0	12,15	8	-0.0248	-0.0208	19.2	0.0116	-0.0010
$\sqrt{CH_4}$	XS_0	13,15	10	-0.0088	-0.0073	20.8	0.0052	0.0051
$\log_2(XCO_2)$	$\sqrt{CH_4}$	8,10	12	-2.8451	-2.9866	4.7	273.7377	0.6210
$\sqrt{CH_4}$	$\log_2(XCO_2)$	8,10	13	-0.3515	-0.3284	7.0	96.2146	0.2037
XS_0	$\log_2(XCO_2)$	12,15	14	-40.2634	-37.1243	8.5	0.4662	-0.0103
XS_0	$\sqrt{CH_4}$	13,15	15	-113.0415	-120.2520	6.0	0.5889	0.7713
$\log_2(XCO_2)$	XS_0	14	8	-0.0269	-0.0208	29.3	-0.0003	-0.0010
$\sqrt{CH_4}$	XS_0	15	10	-0.0083	-0.0073	13.6	0.0064	0.0051
$\log_2(XCO_2)$	$\sqrt{CH_4}$	13	12	-3.0449	-2.9866	0.2	0.6201	0.6210
$\sqrt{CH_4}$	$\log_2(XCO_2)$	12	13	-0.3348	-0.3284	0.2	0.2079	0.2037
XS_0	$\log_2(XCO_2)$	8	14	-48.0087	-37.1243	29.3	-0.0050	-0.0103
XS_0	$\sqrt{CH_4}$	10	15	-136.5883	-120.2520	13.6	0.7032	0.77133

TABLE 1. Comparison between regressed relationships in GISS ModelE2 (Equations 8, 10, 12-15) and recovered equations from substitution or inversion. First two columns describe the independent and dependent variables in the equation. Third column indicates which two equations were combined or which equation was inverted such that the result can be compared with an equation obtained from regression (fourth column). Columns 5 and 8 show the slope and intercept (respectively) of the relation obtained by substitution. Columns 6 and 9 show the slope and intercept of the original relation. Column 7 shows the percent error, calculated as (slope minus original slope)/(original slope). % error is not given for intercepts, as such calculations would be dependent upon a chosen baseline and hence arbitrary. Slope and intercept values are rounded to four decimal places, and percent error is rounded to one decimal place.